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Simplifying superstring action on $AdS_5 \times S^5$

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Abstract: Type IIB string action on $AdS_5 \times S^5$ constructed in hep-th/9805028 is put into a form where it becomes quadratic in fermions. This is achieved by performing 2-d duality (T-duality) on the action in which kappa-symmetry was fixed in the Killing gauge in hep-th/9808038. We discuss some properties and possible applications of the resulting action.

Keywords: D-branes, Conformal Field Models in String Theory, Superstrings and Heterotic Strings.

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Contents

1. Introduction

The classical superstring action of Green-Schwarz type [1] was recently constructed in the non-trivial maximally supersymmetric $D = 10$ type IIB supergravity vacuum [2] (which is also the near horizon space of the D3 brane [3]), i.e. in the $AdS_5 \times S^5$ background [4, 5, 6]. This action has local κ-symmetry and 2-d reparametrization symmetry. By gauge-fixing κ-symmetry one can reduce the number of fermions by 1/2 to match the number of physical bosonic and fermionic degrees of freedom. The gauge-fixing of κ-symmetry was performed in [7] developing the proposal [8] and the action was found which has terms at most quartic in fermions.[1] The special κ-symmetry gauge using the projector parallel to D3-brane directions allowed to substantially reduce the power of fermionic terms in the action. Still, in contrast to the Green-Schwarz action in the light-cone gauge (in flat space or on a group manifold), the resulting action was not quadratic in fermions.

One of the purposes of this paper is to describe a simple transformation that relates the gauge-fixed action to an action which is *quadratic* in fermions. This transformation is the 2-d scalar-scalar duality applied to the string coordinates parallel to the 'D3-brane' directions. The corresponding target space transformation is T-duality along the

[1] A similar action was found in [9] using supersolvable (*Ssolv*) algebra approach [10]. That action will be qualified as a gauge-fixed action if the relation between the gauge-fixing and *Ssolv* algebra approach is clarified. It is not quite clear whether the actions in [7] and in [9] actually agree given also that different choices of bosonic (and fermionic) coordinates were used: Cartesian and horospherical in [7] and projective coordinates on S^5 in [9].

4 world-volume D3-brane directions. It transforms the near-core D3-brane background (i.e. $AdS_5 \times S^5$) into the near-core D-instanton background [11] smeared in the 4 directions. This T-dual background has conformally flat $D = 10$ string-frame metric which, remarkably, is again equivalent to the $AdS_5 \times S^5$ metric (as can be seen by the coordinate transformation $y \to 1/y$ in the radial direction).[2] In addition, there are non-vanishing dilaton and Ramond-Ramond scalar backgrounds.[3]

The resulting dual action (which, as usual, is expected to represent an equivalent 2-d conformal theory, at least in the case of toroidal compactification of D3-brane directions) can thus be interpreted as an action of a fundamental string propagating in the near-core region of the smeared D-instanton background. The drastic simplification of the fermionic part of the action may be related to the fact that the D-instanton background has flat $D = 10$ Einstein-frame metric.

The fermionic part of the action takes the form $\bar{\theta} A \theta$, where A is a first-order differential operator, $A \sim \partial X \partial$. Remarkably, A does not depend on the 2-d metric as the fermionic term is of WZ type. Since the κ-symmetry gauge is already fixed, A should be non-degenerate for generic string background. We shall show that invertibility of the operator A puts certain constraints on the properties of the bosonic string coordinate X background.

Having quadratic fermionic action should be important for solution of the corresponding 2-d conformal model. At the classical level, one may be able to solve the fermionic equations of motion explicitly. At the quantum level, one can integrate out the fermions, obtaining the effective action $S_{eff}(X) = S(X) - \frac{1}{2} \ln \det A(X)$ for the bosonic coordinates X. We expect that the fermionic determinant will induce a WZ-type term (cf. [14]), explaining how the presence of the fermionic couplings representing a non-trivial RR background makes the symmetric space $AdS_5 \times S^5$ sigma model conformal at the quantum level. That would provide a non-perturbative demonstration of the conformal invariance of the $AdS_5 \times S^5$ superstring model, complementing the all-order perturbative conformal invariance proof given in [4, 15]. While the bosonic $AdS_5 \times S^5$ sigma-model is classically integrable but not solvable and not conformal at the quantum level, one may expect that the theory defined by the superstring action or $S_{eff}(X)$ will have properties which are similar to those of the group space sigma model (WZW theory), and, in particular, may be explicitly solvable as a 2-d theory.

One possible application of the superstring action that we shall briefly discuss below is to the computation of the 1-loop correction to the semiclassical value of the Wilson factor in [16]. It is plausible that the Lüscher $1/L$ term in the effective potential is non-vanishing in this case (cf. [17]).

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[2]Note that the near-core string-frame metric of the *localized* (not smeared) D-instanton background is flat [12, 13].

[3]The $AdS_5 \times S^5$ metric can be completed to a solution of type IIB supergravity by either the RR 4-form background or by the T-dual dilaton and RR 0-form background.

2. Superstring action in $AdS_5 \times S^5$ in a κ-symmetry gauge

We start with the classical $AdS_5 \times S^5$ action obtained in the closed form via a supercoset construction [4, 5, 6] ($2\pi\alpha' = 1$)

$$S = -\frac{1}{2}\int d^2\sigma \, \left(\sqrt{-g}\, g^{ij} L_i^{\hat{a}} L_j^{\hat{a}} + 4i\epsilon^{ij}\int_0^1 ds \; L_{is}^{\hat{a}} \mathcal{K}^{IJ}\bar{\Theta}^I\Gamma^{\hat{a}} L_{is}^J\right) . \tag{2.1}$$

Here $\mathcal{K}^{IJ} \equiv \mathrm{diag}(1,-1),\;\; I,J = 1,2$ and $\hat{a} = (a,a') = (0,\ldots,4,5,\ldots,9)$. The invariant 1-forms $L^I = L^I_{s=1},\;\; L^{\hat{a}} = L^{\hat{a}}_{s=1}$ are given by

$$L_s^I = \left(\frac{\sinh(s\mathcal{M})}{\mathcal{M}} D\Theta\right)^I, \qquad L_s^{\hat{a}} = e_{\hat{m}}^{\hat{a}}(X)dX^{\hat{m}} - 4i\bar{\Theta}^I\Gamma^{\hat{a}}\left(\frac{\sinh^2\left(\frac{1}{2}s\mathcal{M}\right)}{\mathcal{M}^2} D\Theta\right)^I, \tag{2.2}$$

where $X^{\hat{m}}$ and Θ^I are the bosonic and fermionic superstring coordinates and

$$\begin{aligned}
(\mathcal{M}^2)^{IL} &= \epsilon^{IJ}(-\gamma^a\Theta^J\bar{\Theta}^L\gamma^a + \gamma^{a'}\Theta^J\bar{\Theta}^L\gamma^{a'}) + \frac{1}{2}\epsilon^{KL}(\gamma^{ab}\Theta^I\bar{\Theta}^K\gamma^{ab} - \gamma^{a'b'}\Theta^I\bar{\Theta}^K\gamma^{a'b'}) , \\
(D\Theta)^I &= \left[d + \frac{1}{4}(\omega^{ab}\gamma_{ab} + \omega^{a'b'}\gamma_{a'b'})\right]\Theta^I - \frac{1}{2}i\epsilon^{IJ}(e^a\gamma_a + ie^{a'}\gamma_{a'})\Theta^J .
\end{aligned} \tag{2.3}$$

The Dirac matrices are split in the '5+5' way, $\Gamma^a = \gamma^a \times 1 \times \sigma_1, \quad \Gamma^{a'} = 1 \times \gamma^{a'} \times \sigma_2$, where σ_k are Pauli matrices (see [4, 6] for details on notation).

Let us review the κ-symmetry gauge fixing of this action performed in [7]. We shall use the 'D3-brane adapted' or '4+6' bosonic coordinates $X^{\hat{m}} = (x^p, y^t)$ in which the $AdS_5 \times S^5$ metric is split into the parts parallel and transverse to the D3-brane directions (we take the radius parameter to be $R = 1$)

$$ds^2 = y^2 dx^p dx^p + \frac{1}{y^2}dy^t dy^t , \qquad y^2 \equiv y^t y^t , \tag{2.4}$$

where $p = 0,\ldots,3,\;\; t = 4,\ldots,9$. In what follows the contractions of the indices p is understood with Minkowski metric and indices t – with Euclidean metric. The κ-symmetry gauge is fixed using the 'parallel to D3-brane' Γ-matrix projector[4]

$$\Theta_-^I = 0 , \qquad \Theta_\pm^I \equiv \mathcal{P}_\pm^{IJ}\Theta^J , \qquad \mathcal{P}_\pm^{IJ} = \frac{1}{2}\left(\delta^{IJ} \pm \Gamma_{0123}\epsilon^{IJ}\right) , \qquad \mathcal{P}_+\mathcal{P}_- = 0 . \tag{2.5}$$

In '5+5' coordinates ($x^a = (x^p, x^4 = y)$ and $\xi^{a'}$ coordinates on S^5) one finds that ($\Gamma_{0123} = i\gamma_4 \times 1 \times 1,\;\; \omega^{p4} = e^p$)

$$(D\Theta)^I = \left[\delta^{IJ}(d + \frac{1}{4}\omega^{a'b'}\gamma_{a'b'}) + \frac{1}{2}\epsilon^{IJ}(e^{a'}\gamma_{a'} - ie^4\gamma_4) + \frac{1}{2}e^p\gamma_p\gamma_4\mathcal{P}_-^{IJ}\right]\Theta^J . \tag{2.6}$$

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[4] This projector is hermitean and anticommutes with Γ^0 so that $\bar{\Theta}_+ = \Theta^\dagger\mathcal{P}_+\Gamma^0 = \bar{\Theta}\mathcal{P}_-,\;\; \bar{\Theta} \equiv \Theta^\dagger\Gamma^0$. For two spinors Θ and Ψ one has $\bar{\Theta}_+^I\Gamma^{\hat{a}}\Psi_+^I = \Theta^\dagger\Gamma^0\mathcal{P}_-\Gamma^{\hat{a}}\mathcal{P}_+\Psi$. Since Γ_{0123} anticommutes with Γ^p, this expression does not vanish only in the 'parallel' directions, i.e. $\bar{\Theta}_+\Gamma^p\Psi_+ \neq 0,\;\; \bar{\Theta}_+\Gamma^t\Psi_+ = 0$. For example, only the 'parallel' part ($\hat{a} = p$) of $\bar{\Theta}_+^I\Gamma^{\hat{a}}\partial\Theta_+^I$ remains in this gauge in the 'kinetic' part of the flat-space superstring action. At the same time, only the 'transverse' part ($\hat{a} = t$) survives in the combination $\mathcal{K}^{IJ}\bar{\Theta}_+^I\Gamma^{\hat{a}}\partial\Theta_+^J$ which appears in the WZ term in the string action.

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Using that the S^5 part of the covariant derivative satisfies $D_5^{IJ} \equiv \delta^{IJ}(d + \frac{1}{4}\omega^{a'b'}\gamma_{a'b'}) + \frac{1}{2}\epsilon^{IJ}e^{a'}\gamma_{a'} = (\Lambda d\Lambda^{-1})^{IJ}$, $(D_5)^2 = 0$, where the spinor matrix $\Lambda^{IJ} = \Lambda^{IJ}(\xi)$ is a function of the S^5 coordinates,[5] one finds that $(D\Theta_+)^I$ can be written as

$$D\Theta_+ = (d - \frac{1}{2}d\log y + \Lambda d\Lambda^{-1})\Theta_+^I = y^{1/2}\Lambda\ d\ (y^{-1/2}\Lambda^{-1}\Theta_+)\,. \tag{2.7}$$

Equation (2.7) suggests to make the change of the fermionic variable $\Theta \to \theta$

$$\Theta_+^I = y^{1/2}\Lambda^{IJ}(\xi)\ \theta_+^J\,, \qquad \mathcal{P}_-^{IJ}\theta_+^J = 0\,, \qquad D\Theta_+^I = y^{1/2}\Lambda d\theta_+^I\,. \tag{2.8}$$

If we further transform from the coordinates $(y, \xi^{a'})$ to the 6-d Cartesian coordinates y^t in (2.4), $y^t = \frac{y}{\sqrt{1+\xi^2}}(1, \xi^{a'})$, that would effectively absorb the matrix Λ into an $SO(6)$ spinor rotation.[6] This simplification is suggested [8] by the form of the Killing spinors in $AdS_5 \times S^5$ space viewed as the near-horizon D3-brane background. In particular, writing the 10-d covariant derivative (including the Lorentz connection and 5-form terms) in the '4+6' coordinates in (2.4) one learns [19] that when acting on the constrained spinor Θ_+ it becomes simply $D\Theta_+ = y^{1/2}d\theta_+$, $\theta_+ \equiv y^{-1/2}\Theta_+$.

As a result, $\mathcal{M}^2 D\Theta_+ = 0$ [7] and the fermionic sector of the action reduces only to terms quadratic and quartic in θ_+^I. Using $\mathcal{P}_-^{IJ}\theta_+^J = 0$ to eliminate θ_+^2 in favour of $\theta_+^1 \equiv \vartheta$, one finds that the κ-symmetry gauge-fixed string action in $AdS_5 \times S^5$ background (2.1) expressed in terms of the bosonic coordinates $X^{\hat{m}} = (x^p, y^t)$ and the *single* $D = 10$ Majorana-Weyl spinor ϑ takes the following simple form[7]

$$S = -\frac{1}{2}\int d^2\sigma\ \Big[\sqrt{-g}\, g^{ij}\left(y^2(\partial_i x^p - 2i\bar\vartheta\Gamma^p\partial_i\vartheta)(\partial_j x^p - 2i\bar\vartheta\Gamma^p\partial_j\vartheta) + \frac{1}{y^2}\partial_i y^t\partial_j y^t\right) + \\ +\ 4i\epsilon^{ij}\partial_i y^t\bar\vartheta\Gamma^t\partial_j\vartheta\ \Big]\,. \tag{2.9}$$

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[5]The same matrix appears in the expression for the Killing spinors on S^5 [18].

[6]This may be interpreted as a rotation from $(y, 0, 0, 0, 0, 0)$ to a generic 6-vector y^t by the $SO(6)$ transformation parametrised by S^5 angles or by the unit vector $\hat{y}^t = \frac{y^t}{y}$. In the Cartesian coordinates $y^t = y\hat{y}^t$ the 6-d part of the covariant derivative has the form $D_6^{IJ} = \delta^{IJ}(d + \frac{1}{2}\gamma_{st}\hat{y}^s d\hat{y}^t) + \frac{1}{2}\epsilon^{IJ}\gamma_t(d\hat{y}^t + \hat{y}^t d\log y)$.

[7]Let us summarize the main steps in the derivation of this action. One way to argue that the string action simplifies in the '4+6' cartesian coordinates is to start with the '5+5' or 'direct product' formulation [4] where one has the underlying superalgebra and supercoset space construction and thus can find the explicit solution [5] for the supervielbein in terms of the $\mathcal{M}^2$ matrix in (2.3) (the existence of such solution is not manifest in the cartesian coordinates where the 10-d metric (2.4) does not look like a direct product). One is then able to argue [8, 7] that after fixing the 'D3-brane' κ-symmetry gauge the supergeometry simplifies so that the vielbein contains θ^2 terms at most. Hence the string action should involve only terms of $n = 0, 2, 4$ power in fermions. The structure of the 'kinetic' term in the action is obvious, while to fix the form of the WZ term in the action it is useful to switch from the '5+5' to the manifest 10-d spinor notation. The 3-form that appears in the WZ term is [4] $\mathcal{H} = L^{\hat{a}} \wedge (\bar{L}^1 \wedge \Gamma^{\hat{a}}dL^1 - \bar{L}^2 \wedge \Gamma^{\hat{a}}dL^2)$ and since in the D3-brane gauge for the cartesian coordinate metric (2.4) $L^I = y^{1/2}d(y^{-1/2}\Theta^I)$ [19, 7] we learn that the bracket in $\mathcal{H}$ is essentially the same as in the flat space case and is non-vanishing only for $\hat{a} = t$, i.e. for the transverse 6-space indices. Using that $L^t = y^{-1}dy^t$ one finishes with the above simple 'flat-space' form of the WZ term.

The $\Theta\Theta\partial X\partial X$ terms representing the coupling to the RR background present [4] in the original action (2.1) are now 'hidden' in the $\bar\vartheta\partial\vartheta\partial X$ terms because of the redefinition made in (2.8).

The same action but without y^2 and $1/y^2$ factors is found by fixing the κ-symmetry gauge (2.5) in the flat-space type IIB Green-Schwarz action. This 'D3-brane' gauge breaks the $SO(1,9)$ Lorentz invariance of the action to $SO(1,3)\times SO(6)$, i.e. distinguishes between the 4 'parallel' and 6 'transverse' coordinates. In particular, only the latter ones (y^t) survive in the WZ term. This special property of the gauge (2.5) turns out to be crucial for the observation below that the fermionic terms in the action can be put in a much simpler *quadratic* form by making 2-d duality transformation of the 'parallel' coordinates x^p.

3. T-dual (2-d dual) form of the action

Let us now perform the 2-d duality transformation of the four x^p coordinates. As usual, this is done by putting the action in the first-order form by introducing the 'momenta' (Lagrange multipliers) P^p_i

$$S = -\frac{1}{2}\int d^2\sigma\, \Big[\sqrt{-g}\, g^{ij}\Big(-\frac{1}{y^2}P^p_iP^p_j + 2P^p_i(\partial_j x^p - 2i\bar\vartheta\Gamma^p\partial_j\vartheta) + \frac{1}{y^2}\partial_i y^t\partial_j y^t\Big) + \\ + 4i\epsilon^{ij}\partial_i y^t\bar\vartheta\Gamma^t\partial_j\vartheta\Big]\,. \tag{3.1}$$

Integrating out x^p and solving the resulting constraint on P^p_i as

$$\sqrt{-g}g^{ij}P^p_j = \epsilon^{ij}\partial_j\tilde x^p\,, \tag{3.2}$$

we finish with the dual action

$$\tilde S = -\frac{1}{2}\int d^2\sigma\Big[\sqrt{-g}g^{ij}\frac{1}{y^2}(\partial_i\tilde x^p\partial_j\tilde x^p + \partial_i y^t\partial_j y^t) + 4i\epsilon^{ij}\bar\vartheta(\partial_i\tilde x^p\Gamma^p + \partial_i y^t\Gamma^t)\partial_j\vartheta\Big]\,. \tag{3.3}$$

At the quantum level, the 2-d duality is accompanied [20] by the dilaton term [21] which should be added to the dual action (3.3) to preserve its conformal invariance,

$$\Delta\tilde S = \frac{1}{4\pi}\int d^2\sigma\sqrt{-g}R^{(2)}\phi(X)\,, \qquad \phi = \phi_0 - 4\ln y\,. \tag{3.4}$$

A remarkable property of the action (3.3) is not only that its fermionic part is quadratic in θ but also that it does not depend on 2-d metric, i.e. is given by a WZ type term. This WZ term is linear in the bosonic coordinates, i.e. has formally the same form as in flat target space. A somewhat surprising conclusion is that adding this fermionic term to the bosonic symmetric space $AdS_5\times S^5$ sigma model action (with dilaton term (3.4) also included) should give a conformally invariant 2-d theory! [8]

[8] Let us mention also that written in terms of the variable $\rho = \frac{1}{2}\ln y$ the bosonic part of the action (3.3),(3.4) is very similar to the one discussed in [22]: the background metric and dilaton are $ds^2 = d\rho^2 + e^{-2\rho}d\tilde x^pd\tilde x^p + d\Omega_5^2$, $\phi = \phi_0 - 8\rho$.

Let us note that if we would start with the '5+5' form of the action in which the WZ term has a more complicated structure depending on the S^5 spinor matrix $\Lambda(\xi)$ in (2.8) (which drops out of the 'kinetic' term in (2.9)), that would not affect the 2-d duality transformation step, and the resulting dual action will still be quadratic in fermions.

The duality has partially restored a 'symmetry' between 'parallel' and 'transverse' coordinates. Starting with the flat-space analogue of (2.9) and performing the same duality transformation one would get (3.3) without the $1/y^2$ factor, i.e. obtain indeed the $SO(1,9)$ invariant action for $\tilde{X}^{\hat{a}} = (\tilde{x}^p, y^t)$ and ϑ

$$\tilde{S}_{flat} = -\frac{1}{2}\int d^2\sigma \left(\sqrt{-g}g^{ij}\partial_i\tilde{X}^{\hat{a}}\partial_j\tilde{X}^{\hat{a}} \ + \ 4i\epsilon^{ij}\partial_i\tilde{X}^{\hat{a}} \ \bar{\vartheta}\Gamma^{\hat{a}}\partial_j\vartheta\right). \qquad (3.5)$$

This looks like the type I (or heterotic) flat-space string action but without the standard fermionic terms complementing ∂X in the 'kinetic' part of the action (as a result, the κ-symmetry is broken, as, of course, should be in the present type IIB gauge-fixed theory).[9] We would arrive at exactly the same flat-space dual action (3.5) had we started with the flat-space IIB action, used the 'Dq-brane' combination $\Gamma_{0\ldots q}$ (q =odd) instead of Γ_{0123} in (2.5) and dualized the 'parallel' coordinates x^p, $p = 0, \ldots, q$. However, that procedure would no longer generalize to curved $AdS_5 \times S^5$ space unless $q = 3$: the form of the $AdS_5 \times S^5$ background (2.4) prefers the 'D3-brane' gauge choice (2.5).

As was already mentioned above, the fact that 2-d duality simplified the structure of the fermionic terms is related to the key property of the gauge-fixed action (2.9) or its flat-space counterpart: only part of the bosonic coordinates ('transverse' ones) appear in the WZ term. For example, the standard type I superstring action which has similar form of a sum of a 'kinetic' (2-d metric dependent) and a WZ term, i.e. $\int(\partial X - \bar{\theta}\partial\theta)^2 + idX \wedge \bar{\theta}d\theta$, *preserves* its form under 2-d duality applied to any of the coordinates $X^{\hat{a}}$, as all of them enter both the first and the second term in that action.

The dual action (3.3) can be interpreted as describing the fundamental string propagating in the background representing the near-core region of the D-instanton smeared in the 4 directions $\tilde{x}^p$. This background is T-dual[10] to the original D3-brane background and has the form

$$ds^2 = H^{1/2}(d\tilde{x}^p d\tilde{x}^p + dy^t dy^t)\,, \qquad e^\phi = H\,, \qquad C = H^{-1}\,, \qquad H = \frac{\tilde{R}^4}{y^4}\,. \qquad (3.6)$$

Note that this conformally flat $D = 10$ string-frame metric is actually equivalent to the $AdS_5 \times S^5$ metric (2.4): changing the coordinates y^t so that the radial coordinate

[9] Note that fixing the light-cone κ-symmetry gauge $\Gamma^+\theta = 0$ in the flat-space IIB action leads to the action ($\int \partial X^+\bar{\vartheta}\Gamma^-\partial\vartheta$) which is similar to (3.5) but is not 10-d Lorentz-invariant and has the fermionic term which depends on the 2-d metric, while in (3.5) the fermionic term has purely 'topological' WZ structure.

[10] Note that T-duality along the isometric D3-brane directions preserves supersymmetry since the Killing spinors do not depend on these isometric coordinates [19].

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gets inverted, $y = 1/y'$, we get (we set $\tilde{R} = 1$ as above)

$$ds^2 = y'^2 d\tilde{x}^p d\tilde{x}^p + \frac{1}{y'^2} dy'^t dy'^t \,, \qquad e^{\phi} = C^{-1} = y'^4 \,. \tag{3.7}$$

Thus, like the action (2.9), the dual action (3.3) can also be *directly* interpreted as describing a superstring propagating in $AdS_5 \times S^5$ space, now supplemented not by the 4-form background but by the dilaton and 0-form backgrounds.

Since the fermionic term in (3.3) does not depend on the 2-d metric, the semiclassically equivalent Nambu-type action obtained by solving for g_{ij} is thus *also* quadratic in θ:

$$\begin{aligned}\tilde{S} = -\frac{1}{2}\int d^2\sigma \, \Big[& \sqrt{-\det[\frac{1}{y^2}(\partial_i \tilde{x}^p \partial_j \tilde{x}^p + \partial_i y^t \partial_j y^t)]} + \\ & + \ 4i\epsilon^{ij}\bar{\vartheta}(\partial_i \tilde{x}^p \Gamma^p + \partial_i y^t \Gamma^t)\partial_j \vartheta \Big] \,. \end{aligned} \tag{3.8}$$

A possible reparametrization gauge choice here may be the static gauge: $\tilde{x}^i = \sigma^i$, $i = 0, 1$, leading to the free 'kinetic' $\bar{\vartheta}\partial\vartheta$ term in the action ($\pi = 2, 3$)

$$\begin{aligned}\tilde{S} = -\frac{1}{2}\int d^2\sigma \, \Big[& \frac{1}{y^2}\sqrt{-\det(\eta_{ij} + \partial_i \tilde{x}^\pi \partial_j \tilde{x}^\pi + \partial_i y^t \partial_j y^t)} + \\ & + \ 4i\epsilon^{ij}\bar{\vartheta}\Gamma_i \partial_j \vartheta + 4i\epsilon^{ij}\bar{\vartheta}(\partial_i \tilde{x}^\pi \Gamma^\pi + \partial_i y^t \Gamma^t)\partial_j \vartheta \Big] \,. \end{aligned} \tag{3.9}$$

The semiclassical expansion is then developed by starting with a particular solution for the string coordinates (e.g., $y = y(\sigma)$, $\tilde{x}^\pi = 0$ as in [16], see below) and integrating over small fluctuations near it.

Let us note that in fixing the static gauge one assumes that the ground state of the string is massive, i.e. this gauge is appropriate for a solitonic string or wound string state but cannot be used to describe a spectrum of a fundamental string in the zero-winding sector. In flat space an adequate gauge for the latter purpose is the combination of the conformal gauge $\sqrt{-g}g^{ij} = \eta^{ij}$ and the light-cone gauge $x^+ = \sigma^0$. However, fixing the light-cone gauge in a curved space which is not a direct product $R^{1,1} \times M^{D-2}$ may not always be possible (see, e.g., [23]). Indeed, in contrast to what happens in flat space (or in a plane-wave background [24]), in the present $AdS_5 \times S^5$ case the conformal-gauge bosonic string equations of motion for x^0, x^1 corresponding to the metric (2.4) $\partial^i(y^2\partial_i)x^p = 0$ do not have $x^+ = \sigma^0$ as a solution for a generic classical configuration of y^t with $y = |y|$ satisfying $\partial^i\partial_i \log y + y^2 \partial^i x^p \partial_i x^p = 0$.

4. Conditions of invertibility of the fermionic kinetic operator

In order to achieve a better understanding of the κ-symmetry gauge choice in (2.5) it is useful to study the issue of invertibility of the fermionic kinetic operator in the actions

(2.9),(3.3). In particular, we shall consider the flat space case obtained by omitting (or just treating as constant) the y^2 and $1/y^2$ factors in the metrics (2.4),(3.6) and the actions (2.9),(3.3). We shall choose the conformal gauge $\sqrt{-g}g^{ij} = \eta^{ij}$. In general, the constraints coming from the equation of motion for the 2-d metric can be written in terms of the vielbein components of the 'momentum' $\Pi_i^{\hat{a}}$ defined by the g_{ij}-dependent part of the action which does not include the WZ term ($z, \bar{z} = \sigma \pm \tau,\ \sigma^0 \equiv \tau,\ \sigma^1 \equiv \sigma$)

$$\Pi_z \cdot \Pi_z \equiv \Pi_z^p \Pi_z^p + \Pi_z^t \Pi_z^t = 0\,, \qquad \Pi_{\bar{z}} \cdot \Pi_{\bar{z}} \equiv \Pi_{\bar{z}}^p \Pi_{\bar{z}}^p + \Pi_{\bar{z}}^t \Pi_{\bar{z}}^t = 0 \tag{4.1}$$

Dots stand for the fermionic terms in the constraints and as above the indices p are contracted with 4-d Minkowski metric and the indices t – with 6-d Euclidean metric. In the case of the action (2.9) $\Pi_i^p = y(\partial_i x^p - 2i\bar{\vartheta}\Gamma^p \partial_j \vartheta)$, $\Pi_i^t = y^{-1}\partial_i y^t$.

Before κ-symmetry gauge fixing the quadratic fermionic terms in the flat-space GS action are

$$\bar{\theta}^1 (\Pi \cdot \Gamma)_z \partial_{\bar{z}} \theta^1 \equiv \bar{\theta}^1 A_1 \theta^1\,, \qquad \bar{\theta}^2 (\Pi \cdot \Gamma)_{\bar{z}} \partial_z \theta^2 \equiv \bar{\theta}^2 A_2 \theta^2\,. \tag{4.2}$$

On the classical equations and constraints we get $(A_1)^2 = (A_2)^2 = 0$, i.e. the fermionic operator is degenerate for any classical string background. As we shall see below, after the κ-symmetry gauge fixing in (2.5) the degeneracy is removed provided the background is constrained in a certain way (e.g., to have non-zero Π_σ^2 for vanishing Π_τ^2 for massless 10-d states).

The quadratic term in the fermionic part of the gauge-fixed action (2.9) is (we omit the fermionic terms in Π)

$$\bar{\vartheta}\ y\,[(\Pi \cdot \Gamma)_z \partial_{\bar{z}} + (\Pi * \Gamma)_{\bar{z}} \partial_z]\ \vartheta \ \equiv\ \bar{\vartheta}\ A\ \vartheta\,, \tag{4.3}$$

where we introduced the notation

$$\Pi_i^p \Gamma^p + \Pi_i^t \Gamma^t = (\Pi \cdot \Gamma)_i\,, \qquad \Pi_i^p \Gamma^p - \Pi_i^t \Gamma^t = (\Pi * \Gamma)_i\,. \tag{4.4}$$

Using the equations of motion for $X^{\hat{m}}$ and the constraints (4.1), the square of the kinetic operator A can be written as

$$\begin{aligned} A^2 &= y^2[(\Pi \cdot \Gamma)_z (\Pi * \Gamma)_{\bar{z}} + (\Pi * \Gamma)_{\bar{z}} (\Pi \cdot \Gamma)_z] \partial_z \partial_{\bar{z}} + \cdots \\ &= y^2 [\Pi_z^p \Pi_{\bar{z}}^p - \Pi_z^t \Pi_{\bar{z}}^t] \partial_z \partial_{\bar{z}} + \cdots\,, \end{aligned} \tag{4.5}$$

where dots stand for lower-derivative ∂y-dependent terms which are absent in the flat space limit ($y = \text{const}$). In flat space A^2 is invertible even on massless ($M_{10}^2 = 0$) 10-d string states with $(\Pi \cdot \Pi)_\tau = 0$ and $(\Pi \cdot \Pi)_\sigma = 0$ if the $X^{\hat{m}}$ background is a BPS one,

$$(\Pi^p \Pi^p)_\tau = -M_4^2\,, \qquad (\Pi^t \Pi^t)_\tau = Z^2\,, \qquad M_4^2 = Z^2\,, \tag{4.6}$$

$$A^2 = -2y^2 M_4^2 \partial_z \partial_{\bar{z}} = -2y^2 Z^2 \partial_z \partial_{\bar{z}}\,. \tag{4.7}$$

In the case of the dual action (3.3) the constraints are expressed in terms of $\Pi^p_i = y^{-1}\partial_i \tilde{x}^p$, and $\Pi^t_i = y^{-1}\partial_i y \Gamma^t \partial_j \vartheta$, i.e. do not include fermionic terms. In the flat space limit (i.e. ignoring derivatives of y) the square of the corresponding fermionic kinetic operator takes the form

$$\tilde{A}^2 = y^2 \Pi^2 \partial_z \partial_{\bar{z}}\,, \qquad \Pi^2 \equiv \eta_{\hat{a}\hat{b}} \Pi^{\hat{a}}_z \Pi^{\hat{b}}_{\bar{z}}\,. \tag{4.8}$$

Thus for the invertibility of $\tilde{A}^2$ we need $\Pi^2 = -(\Pi\cdot\Pi)_\tau + (\Pi\cdot\Pi)_\sigma = 2(\Pi\cdot\Pi)_\sigma \neq 0$, where we have used the constraint $(\Pi\cdot\Pi)_\tau + (\Pi\cdot\Pi)_\sigma = 0$.

Similar conclusion is reached in the curved space case. In particular, it is possible to show that the fermionic operator $\tilde{A}^2$ is degenerate on a general center-of-mass (σ-independent) solution of the bosonic equations and constraints that follow from the dual action (3.3). Indeed, if we assume that $\partial_\sigma \tilde{x}^p = 0$, $\partial_\sigma y^t = 0$, then $\partial_\tau \tilde{x}^p = v^p y^2$, $\partial_\tau y^t = u^t y^2$, where v^p, u^t are constant vectors subject to the constraint

$$v^p v^p + u^t u^t = 0\,. \tag{4.9}$$

As a result,[11]

$$\tilde{x}^p = a^p + v^p f(\tau)\,, \qquad y^t = b^t + u^t f(\tau)\,, \tag{4.10}$$

$$f(\tau) = -\frac{b\cdot u}{u^2} + \frac{\omega}{u^2}\tan\omega(\tau-\tau_0)\,, \qquad \omega^2 \equiv b^2u^2 - (b\cdot u)^2, \tag{4.11}$$

where the constants $a^p, v^p, b^t, u^t, \tau_0$ are functions of the initial data $\tilde{x}^p(0), \partial_\tau\tilde{x}^p(0)$, $y^t(0), \partial_\tau y^t(0)$. [12] The fermionic term in (3.3) is thus proportional to $y^2(\tau)\ \bar{\vartheta}(v^p\Gamma^p + u^t\Gamma^t)\partial_\sigma\vartheta$, and is degenerate ($\tilde{A}^2 = 0$) since $(v^p\Gamma^p + u^t\Gamma^t)^2 = 0$ as follows from (4.9). To have a non-degenerate fermionic operator one has to consider inhomogeneous (σ-dependent) string configurations.

5. Application to Wilson loop calculation

A particular class of such static but σ-dependent configurations is, in fact, of physical interest in connection with Wilson loop calculations in [16]. Let us now comment on the application of the $AdS_5 \times S^5$ superstring action (2.9) to this problem. The aim is to compute the string path integral $e^{-W} = \int [dxd\theta] e^{-S}$ by expanding near a particular classical string solution. The action S may be taken in the Nambu form (i.e. with g_{ij} eliminated)[13] and one may fix the static gauge $x^0 = \sigma^0$, $x^1 = \sigma^1$, where σ^i are the (Euclidean) world-sheet coordinates which run from 0 to T and $-L/2$ to $L/2$. Following

[11] Similar solution of center-of-mass string (i.e. massless particle) equations in $AdS_n \times S^n$ was found by R.R. Metsaev (unpublished).

[12] Note that y^t depends only on part of b^t which is transverse to u^t, i.e. $b^t - \frac{b\cdot u}{u^2}u^t$. The parameter τ_0 compensates for the 'missing' longitudinal component of b^t and can be determined from the constraints $u^t y^t(\tau_0) = 0$ or $y^t(0)\partial_\tau y^t(0) = -\frac{\omega^3}{u^2}\frac{\sin\omega\tau_0}{\cos^3\omega\tau}$.

[13] The two forms of the string action are equivalent in this context of semiclassical expansion [25].

[16], let us consider a string solution which is static and has only the radial coordinate $y = \sqrt{y^t y^t}$ changing with σ^1. The bosonic part of string action is then proportional to $T \int d\sigma^1 \sqrt{(\partial_1 y)^2 + y^4}$. The stationary point is determined by the equation

$$(\partial_1 y)^2 + y^4 = a y^8 \,, \qquad a = \text{const} \,, \tag{5.1}$$

with the boundary condition that y takes the minimal value $y_* = a^{-1/4}$ at $\sigma = 0$ and runs to infinity at $-L/2$ and $L/2$. While the solution $y(\sigma^1)$ does not have a simple analytic form, (5.1) allows to eliminate $\partial_1 y$ in terms of y. The classical value of the string action is proportional to T/L. Expanding the action near the solution one may compute the 1-loop correction to the effective potential.

Here we shall consider only the quadratic fermionic part as it follows from the Nambu analogue of the gauge-fixed action (2.9). Using (5.1), one finds that the classical value of the induced Euclidean 2-d metric is $g_{ij} = y^2 \delta_{ij} + \frac{1}{y^2} \partial_i y \partial_j y = \text{diag}(y^2, a y^6)$. Thus $\sqrt{g} g^{ij} = \text{diag}(\sqrt{a} y^2, \frac{1}{\sqrt{a} y^2})$, and the sum of the quadratic fermionic terms in the action takes the form

$$\begin{aligned} S_F &= 2i \int d^2\sigma \, \left(\sqrt{g} g^{ij} y^2 \bar\vartheta \Gamma_i \partial_j \vartheta - \epsilon^{ij} \partial_i y^t \bar\vartheta \Gamma^t \partial_j \vartheta \right) \\ &= 2i \int d^2\sigma \, \bar\vartheta \Big[\frac{1}{\sqrt{a}} \Gamma^1 \partial_1 + (\sqrt{a} y^4 \Gamma^0 + \partial_1 y \Gamma^y) \partial_0 \Big] \vartheta \,. \end{aligned} \tag{5.2}$$

We used the fact that in the static gauge $\partial_i x^p = \delta_i^p$ and took into account that only the radial part of y^t has a non-trivial σ^1-dependent background value (the radial component Γ^y depends on constant angular parameters). The resulting fermionic operator is non-degenerate, and it may be possible to compute the contribution of its determinant to W in the leading order in large L, T.

In flat space (or for a y=const solution) one finds that the fluctuations of $d-2=8$ transverse bosonic coordinates (which are periodic in σ^0 and σ^1) give [26] $W = \frac{1}{2}(d-2) \log\det(-\partial^2) = -(d-2)\frac{\pi T}{24 L}$. In the flat-space superstring case this contribution is cancelled by the contribution of the fermionic determinant: the total effective number of transverse world-sheet degrees of freedom is equal to zero because of supersymmetry. It is plausible that the 1-loop $1/L$ correction to the effective potential will not, however, vanish in the present curved space case as there is no reason to expect that the solution $y = y(\sigma^1) \neq$ const should preserve effective world-sheet supersymmetry.

6. Conclusions

To conclude, we have shown that choosing the 'D3-brane' or '4-d space-time' adapted κ-symmetry gauge in the $AdS_5 \times S^5$ superstring action and duality-rotating the four isometric space-time coordinates one obtains an action in which the fermionic term is quadratic and does not depend on the world-sheet metric. The '4+6' Cartesian parametrisation of the 10-d space thus led to a substantial simplification of the fermionic sector of the theory. This should hopefully allow one to make progress towards extracting more non-trivial information about the $AdS_5 \times S^5$ string theory and thus about its dual [27] – $N = 4$ super Yang-Mills theory.

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